CLAIMS

1. An apparatus for blind separation of an overcomplete set of mixed signals, the apparatus comprising:

   i. a data processing system including an input for receiving mixed signals from a plurality of sensors configured to receive mixed signal samples comprising a mixture of signals transmitted from signal sources through an environment and noise, a signal processor attached with the input for receiving the mixed signals from the sensors, and a memory for storing data during operations of the signal processor; the data processing system further comprising:

   ii. means for storing data representing the input from the sensors in a mixed signal matrix **X**;

   iii. means for storing data representing the noise in a noise matrix **V**;

   iv. means for storing data representing an estimate of the individual signals from the mixture of signals from the signal sources in a source signal estimate matrix $\hat{\mathbf{S}}$;

   v. means for storing data representing an estimate of the effects of the environment in a estimated mixing matrix $\hat{\mathbf{A}}$ where the matrices are related by $\mathbf{X} = \hat{\mathbf{A}}\hat{\mathbf{S}} + \mathbf{V}$;

   vi. means for generating an initial estimate of the estimated mixing matrix $\hat{\mathbf{A}}$;

   vii. means for determining the number of signal sources and associated lines of correlation of each of the signal sources from the estimated mixing matrix $\hat{\mathbf{A}}$, and for representing the signal sources in the source signal estimate matrix $\hat{\mathbf{S}}$;

   viii. means for jointly optimizing the source signal estimate matrix $\hat{\mathbf{S}}$ and the estimated mixing matrix $\hat{\mathbf{A}}$ in an iterative manner, to generate an optimized source signal estimate matrix $\hat{\mathbf{S}}$ and a final estimated mixing matrix $\hat{\mathbf{A}}$; and

   ix. means for restoring the separated source signals from the optimized source signal estimate matrix $\hat{\mathbf{S}}$, whereby a plurality of mixed signals from unknown sources traveling through an environment with added noise may be separated so that the original, separate signals may be reconstructed.

2. An apparatus for blind separation of an overcomplete set of mixed signals as set forth in claim 1, wherein the means for generating an initial estimate of the estimated mixing matrix $\hat{\mathbf{A}}$ comprises:

i. means for transforming the mixed signal matrix $\mathbf{X}$ into the sparse domain using a transform operator;

ii. means for determining a frequency band within the sparse domain that contains the most information that can be used to determine lines of correlation to determine the number of signal sources;

iii. means for determining a measure and an optimal threshold for the measure for the determination of noise within the frequency band;

iv. means for recalculating the measure used in the determination of the noise within the frequency band using the optimal threshold; and

v. means for determining the local maxima of a distribution of the measure, where the local maxima represent angles which are inserted into the estimated mixing matrix $\hat{\mathbf{A}}$ to provide an initial estimate of the estimated mixing matrix $\hat{\mathbf{A}}$.

3. An apparatus for blind separation of an overcomplete set of mixed signals as set forth in claim 2, wherein the means for jointly optimizing the source signal estimate matrix $\hat{\mathbf{S}}$ and the estimated mixing matrix $\hat{\mathbf{A}}$ in an iterative manner, to generate an optimized source signal estimate matrix $\hat{\mathbf{S}}$ and a final estimated mixing matrix $\hat{\mathbf{A}}$ comprises:

i. means for clustering the mixed signal samples using a geometric constraint; and

ii. means for evaluating a convergence criteria based on the clustered mixed signal samples to determine whether the convergence criteria are met, and if the convergence criteria are not met, iteratively adjusting the clustering of the mixed signal samples and parameters of the geometric constraint to create a new set of clusters until the convergence criteria are met, to provide a final estimated mixing matrix $\hat{\mathbf{A}}$.

4. An apparatus for blind separation of an overcomplete set of mixed signals as set forth in claim 3, wherein the means for transforming the mixed signal matrix **X** into the sparse domain using a transform operator is a Fourier transform operator such that the estimated mixing matrix $\hat{\mathbf{A}}$ is represented in the Fourier transform, and wherein the means for obtaining a multi-band sparse domain estimate of the source signal estimate matrix $\hat{\mathbf{S}}$ using the relationship $\mathbf{X} = \hat{\mathbf{A}}\hat{\mathbf{S}} + \mathbf{V}$ uses a wavelet transform operator to obtain the multi-band sparse domain estimate.

5. An apparatus for blind separation of an overcomplete set of mixed signals as set forth in claim 2, wherein the means for jointly optimizing the source signal estimate matrix $\hat{\mathbf{S}}$ and the estimated mixing matrix $\hat{\mathbf{A}}$ in an iterative manner, to generate an optimized source signal estimate matrix $\hat{\mathbf{S}}$ and a final estimated mixing matrix $\hat{\mathbf{A}}$ further comprises:
   i. means for obtaining a multi-band sparse domain estimate of the source signal estimate matrix $\hat{\mathbf{S}}$ using the relationship $\mathbf{X} = \hat{\mathbf{A}}\hat{\mathbf{S}} + \mathbf{V}$; and
   ii. means for using the adjusted geometric constraint corresponding to the final estimated mixing matrix $\hat{\mathbf{A}}$ in each of the bands of the sparse domain for the source signal estimate matrix $\hat{\mathbf{S}}$ and determining whether a convergence criteria is met for the source signal estimate matrix $\hat{\mathbf{S}}$, and if the convergence criteria are not met, iteratively adjusting the clustering of the mixed signal samples to create a new set of clusters until the convergence criteria are met, to provide a final source signal estimate matrix $\hat{\mathbf{S}}$.

6. An apparatus for blind separation of an overcomplete set of mixed signals as set forth in claim 5, wherein the means for transforming the mixed signal matrix **X** into the sparse domain using a transform operator is a Fourier transform operator such that the estimated mixing matrix $\hat{\mathbf{A}}$ is represented in the Fourier transform, and wherein the means for obtaining a multi-band sparse domain estimate of the source signal

estimate matrix $\hat{\mathbf{S}}$ using the relationship $\mathbf{X} = \hat{\mathbf{A}}\hat{\mathbf{S}} + \mathbf{V}$ uses a wavelet transform operator to obtain the multi-band sparse domain estimate.

7. An apparatus for blind separation of an overcomplete set of mixed signals as set forth in claim 1, wherein the means for generating an initial estimate of the estimated mixing matrix $\hat{\mathbf{A}}$ comprises:
    i. means for transforming the mixed signal matrix **X** into the frequency domain using a Fourier operator;
    ii. means for using a mutual information criterion to determine a frequency band within the sparse domain that contains the most information that can be used to determine lines of correlation to determine the number of signal sources;
    iii. means for determining a random variable $ang = \arctan\frac{x_i(band)}{x_j(band)}$, where $x_i(band)$ and $x_j(band)$ represent Fourier values of mixture in the selected frequency band, and an optimal threshold ANG for *ang*, where the optimal threshold ANG is determined by computing the entropy E(*ang*, ANG) vs. ANG and searching for the optimal value of ANG corresponding to the minimum rate of descent of the entropy E(*ang*, ANG);
    iv. means for recalculating *ang* based on the optimal threshold ANG;
    v. means for using a standard peak detection technique to determine the number and values of local maxima of a histogram of *ang* where the local maxima represent angles which are inserted into the estimated mixing matrix $\hat{\mathbf{A}}$ to provide an initial estimate of the estimated mixing matrix $\hat{\mathbf{A}}$.

8. An apparatus for blind separation of an overcomplete set of mixed signals as set forth in claim 7, wherein the means for jointly optimizing the source signal estimate matrix $\hat{\mathbf{S}}$ and the estimated mixing matrix $\hat{\mathbf{A}}$ in an iterative manner, to generate an optimized source signal estimate matrix $\hat{\mathbf{S}}$ and a final estimated mixing matrix $\hat{\mathbf{A}}$ comprises:

i. means for clustering the mixed signal samples in the Fourier domain along the lines of correlation with one cluster per source using a straight distance metric geometric constraint, with the clusters representing estimates of the Fourier domain representation of $\hat{\mathbf{S}}$, $\mathbf{F}(\hat{\mathbf{S}})$, where $\mathbf{F}$ represents a Fourier domain operator ; and

ii. means for evaluating a convergence criteria, min $\boldsymbol{\lambda}\mathbf{c}^{\mathrm{T}}\left|\mathbf{F}(\hat{\mathbf{S}})\right|$, with the convergence criteria, min $\boldsymbol{\lambda}\mathbf{c}^{\mathrm{T}}\left|\mathbf{F}(\hat{\mathbf{S}})\right|$, developed from the log likelihood function $\mathbf{L}(\mathbf{F}(\hat{\mathbf{S}}) \mid \mathbf{F}(\mathbf{X}), \mathbf{A})$ with the assumption of Laplanicity of source signals in the Fourier domain following the probability $\mathbf{P}(\mathbf{F}(\mathbf{S})) = \frac{\boldsymbol{\lambda}}{\mathbf{2}} e^{-\lambda \mathbf{c}^{\mathrm{T}}\left|\mathbf{F}(\hat{\mathbf{S}})\right|}$, where $\mathbf{c}^{\mathrm{T}} = [1, 1, \ldots 1]$ is a unit vector, with the convergence criteria, min $\boldsymbol{\lambda}\mathbf{c}^{\mathrm{T}}\left|\mathbf{F}(\hat{\mathbf{S}})\right|$, evaluated based on the clustered mixed signal samples to determine whether the convergence criteria, min $\boldsymbol{\lambda}\mathbf{c}^{\mathrm{T}}\left|\mathbf{F}(\hat{\mathbf{S}})\right|$, is met, and if the convergence criteria, min $\boldsymbol{\lambda}\mathbf{c}^{\mathrm{T}}\left|\mathbf{F}(\hat{\mathbf{S}})\right|$, is not met, iteratively adjusting the clustering of the mixed signal samples and parameters of the geometric constraint to create a new set of clusters until the convergence criteria, min $\boldsymbol{\lambda}\mathbf{c}^{\mathrm{T}}\left|\mathbf{F}(\hat{\mathbf{S}})\right|$, is met, to provide a final estimated mixing matrix $\hat{\mathbf{A}}$.

9. An apparatus for blind separation of an overcomplete set of mixed signals as set forth in claim 8, wherein the means for jointly optimizing the source signal estimate matrix $\hat{\mathbf{S}}$ and the estimated mixing matrix $\hat{\mathbf{A}}$ in an iterative manner, to generate an optimized source signal estimate matrix $\hat{\mathbf{S}}$ and a final estimated mixing matrix $\hat{\mathbf{A}}$ further comprises:

i. means for obtaining a multi-band sparse domain estimate of the source signal estimate matrix $\hat{\mathbf{S}}$ using the relationship $\mathbf{X} = \hat{\mathbf{A}}\hat{\mathbf{S}} + \mathbf{V}$, applied in the Wavelet domain; and

ii. means for using the adjusted geometric constraint corresponding to the final estimated mixing matrix $\hat{\mathbf{A}}$ in each of the bands of the Wavelet domain for the source signal estimate matrix $\hat{\mathbf{S}}$, $\mathbf{W}(\hat{\mathbf{S}})$, and determining whether a convergence criteria, min $\lambda\mathbf{c}^{\mathrm{T}}\left|\mathbf{W}(\hat{\mathbf{S}})\right|$ is met for the source signal estimate matrix $\hat{\mathbf{S}}$, where the convergence criteria, min $\lambda\mathbf{c}^{\mathrm{T}}\left|\mathbf{W}(\hat{\mathbf{S}})\right|$, is developed from the log likelihood function $\mathbf{L}(\mathbf{W}(\hat{\mathbf{S}}) \mid \mathbf{W}(\mathbf{X}), \mathbf{A})$ with the assumption of Laplanicity of source signals in the Wavelet domain following the probability $\mathbf{P}(\mathbf{W}(\mathbf{S})) = \frac{\lambda}{2} e^{-\lambda\mathbf{c}^{\mathrm{T}}\left|\mathbf{W}(\hat{\mathbf{S}})\right|}$, where $\mathbf{c}^{\mathrm{T}} = [1, 1, \ldots 1]$ is a unit vector, and if the convergence criteria is not met, min $\lambda\mathbf{c}^{\mathrm{T}}\left|\mathbf{W}(\hat{\mathbf{S}})\right|$, iteratively adjusting the clustering of the mixed signal samples to create a new set of clusters until the convergence criteria, min $\lambda\mathbf{c}^{\mathrm{T}}\left|\mathbf{W}(\hat{\mathbf{S}})\right|$, is met, to provide a final source signal estimate matrix $\hat{\mathbf{S}}$.

10. An apparatus for blind separation of an overcomplete set of mixed signals as set forth in claim 9, wherein the apparatus is configured for separating mixed acoustic signals.

11. An apparatus for blind separation of an overcomplete set of mixed signals as set forth in claim 9, wherein the apparatus is configured for separating mixed radio frequency signals.

12. A method for blind separation of an overcomplete set of mixed signals, using a data processing system including an input for receiving mixed signals from a plurality of sensors configured to receive mixed signal samples comprising a mixture of signals transmitted from signal sources through an environment and noise, a signal processor attached with the input for receiving the mixed signals from the sensors, and a memory for storing data during operations of the signal processor the method comprising the steps of:

i. storing data representing the input from the sensors in a mixed signal matrix $\mathbf{X}$;

ii. storing data representing the noise in a noise matrix $\mathbf{V}$;

iii. storing data representing an estimate of the individual signals from the mixture of signals from the signal sources in a source signal estimate matrix $\hat{\mathbf{S}}$;

iv. storing data representing an estimate of the effects of the environment in a estimated mixing matrix $\hat{\mathbf{A}}$ where the matrices are related by $\mathbf{X} = \hat{\mathbf{A}}\hat{\mathbf{S}} + \mathbf{V}$;

v. generating an initial estimate of the estimated mixing matrix $\hat{\mathbf{A}}$;

vi. determining the number of signal sources and associated lines of correlation of each of the signal sources from the estimated mixing matrix $\hat{\mathbf{A}}$, and for representing the signal sources in the source signal estimate matrix $\hat{\mathbf{S}}$;

vii. jointly optimizing the source signal estimate matrix $\hat{\mathbf{S}}$ and the estimated mixing matrix $\hat{\mathbf{A}}$ in an iterative manner, to generate an optimized source signal estimate matrix $\hat{\mathbf{S}}$ and a final estimated mixing matrix $\hat{\mathbf{A}}$; and

viii. restoring the separated source signals from the optimized source signal estimate matrix $\hat{\mathbf{S}}$, whereby a plurality of mixed signals from unknown sources traveling through an environment with added noise may be separated so that the original, separate signals may be reconstructed.

13. A method for blind separation of an overcomplete set of mixed signals as set forth in claim 12, wherein the step of generating an initial estimate of the estimated mixing matrix $\hat{\mathbf{A}}$ comprises the sub-steps of:

i. transforming the mixed signal matrix $\mathbf{X}$ into the sparse domain using a transform operator;

ii. determining a frequency band within the sparse domain that contains the most information that can be used to determine lines of correlation to determine the number of signal sources;

iii. determining a measure and an optimal threshold for the measure for the determination of noise within the frequency band;

iv. recalculating the measure used in the determination of the noise within the frequency band using the optimal threshold; and

v. determining the local maxima of a distribution of the measure, where the local maxima represent angles which are inserted into the estimated mixing matrix $\hat{\mathbf{A}}$ to provide an initial estimate of the estimated mixing matrix $\hat{\mathbf{A}}$.

14. A method for blind separation of an overcomplete set of mixed signals as set forth in claim 13, wherein the step of jointly optimizing the source signal estimate matrix $\hat{\mathbf{S}}$ and the estimated mixing matrix $\hat{\mathbf{A}}$ in an iterative manner, to generate an optimized source signal estimate matrix $\hat{\mathbf{S}}$ and a final estimated mixing matrix $\hat{\mathbf{A}}$ comprises the sub-steps of:

    i. clustering the mixed signal samples using a geometric constraint; and

    ii. evaluating a convergence criteria based on the clustered mixed signal samples to determine whether the convergence criteria are met, and if the convergence criteria are not met, iteratively adjusting the clustering of the mixed signal samples and parameters of the geometric constraint to create a new set of clusters until the convergence criteria are met, to provide a final estimated mixing matrix $\hat{\mathbf{A}}$.

15. A method for blind separation of an overcomplete set of mixed signals as set forth in claim 14, wherein the step of transforming the mixed signal matrix **X** into the sparse domain using a transform operator is a Fourier transform operation such that the estimated mixing matrix $\hat{\mathbf{A}}$ is represented in the Fourier transform, and wherein the step of obtaining a multi-band sparse domain estimate of the source signal estimate matrix $\hat{\mathbf{S}}$ using the relationship $\mathbf{X} = \hat{\mathbf{A}}\hat{\mathbf{S}} + \mathbf{V}$ uses a wavelet transform operator to obtain the multi-band sparse domain estimate.

16. A method for blind separation of an overcomplete set of mixed signals as set forth in claim 13, wherein the wherein the step of jointly optimizing the source signal estimate matrix $\hat{\mathbf{S}}$ and the estimated mixing matrix $\hat{\mathbf{A}}$ in an iterative manner, to generate an optimized source signal estimate matrix $\hat{\mathbf{S}}$ and a final estimated mixing matrix $\hat{\mathbf{A}}$ further comprises the sub steps of:
    i. obtaining a multi-band sparse domain estimate of the source signal estimate matrix $\hat{\mathbf{S}}$ using the relationship $\mathbf{X} = \hat{\mathbf{A}}\hat{\mathbf{S}} + \mathbf{V}$; and
    ii. using the adjusted geometric constraint corresponding to the final estimated mixing matrix $\hat{\mathbf{A}}$ in each of the bands of the sparse domain for the source signal estimate matrix $\hat{\mathbf{S}}$ and determining whether a convergence criteria is met for the source signal estimate matrix $\hat{\mathbf{S}}$, and if the convergence criteria are not met, iteratively adjusting the clustering of the mixed signal samples to create a new set of clusters until the convergence criteria are met, to provide a final source signal estimate matrix $\hat{\mathbf{S}}$.

17. A method for blind separation of an overcomplete set of mixed signals as set forth in claim 16, wherein the step of transforming the mixed signal matrix **X** into the sparse domain using a transform operator is a Fourier transform operation such that the estimated mixing matrix $\hat{\mathbf{A}}$ is represented in the Fourier transform, and wherein the step of obtaining a multi-band sparse domain estimate of the source signal estimate matrix $\hat{\mathbf{S}}$ using the relationship $\mathbf{X} = \hat{\mathbf{A}}\hat{\mathbf{S}} + \mathbf{V}$ uses a wavelet transform operator to obtain the multi-band sparse domain estimate.

18. A method for blind separation of an overcomplete set of mixed signals as set forth in claim 12, wherein the step of generating an initial estimate of the estimated mixing matrix $\hat{\mathbf{A}}$ comprises the sub steps of:
    i. transforming the mixed signal matrix **X** into the frequency domain using a Fourier operator;

ii. using a mutual information criterion to determine a frequency band within the sparse domain that contains the most information that can be used to determine lines of correlation to determine the number of signal sources;

iii. determining a random variable $ang = \arctan\frac{x_i(band)}{x_j(band)}$, where $x_i(band)$ and $x_j(band)$ represent Fourier values of mixture in the selected frequency band, and an optimal threshold ANG for *ang*, where the optimal threshold ANG is determined by computing the entropy E(*ang*, ANG) vs. ANG and searching for the optimal value of ANG corresponding to the minimum rate of descent of the entropy E(*ang*, ANG);

iv. recalculating *ang* based on the optimal threshold ANG;

v. using a standard peak detection technique to determine the number and values of local maxima of a histogram of *ang* where the local maxima represent angles which are inserted into the estimated mixing matrix $\hat{\mathbf{A}}$ to provide an initial estimate of the estimated mixing matrix $\hat{\mathbf{A}}$.

19. A method for blind separation of an overcomplete set of mixed signals as set forth in claim 18, wherein the step of jointly optimizing the source signal estimate matrix $\hat{\mathbf{S}}$ and the estimated mixing matrix $\hat{\mathbf{A}}$ in an iterative manner, to generate an optimized source signal estimate matrix $\hat{\mathbf{S}}$ and a final estimated mixing matrix $\hat{\mathbf{A}}$ comprises the sub steps of:

i. clustering the mixed signal samples in the Fourier domain along the lines of correlation with one cluster per source using a straight distance metric geometric constraint, with the clusters representing estimates of the Fourier domain representation of $\hat{\mathbf{S}}$, $\mathbf{F}(\hat{\mathbf{S}})$, where **F** represents a Fourier domain operator ; and

ii. evaluating a convergence criteria, min $\boldsymbol{\lambda}\mathbf{c}^{\mathrm{T}}\left|\mathbf{F}(\hat{\mathbf{S}})\right|$, with the convergence criteria, min $\boldsymbol{\lambda}\mathbf{c}^{\mathrm{T}}\left|\mathbf{F}(\hat{\mathbf{S}})\right|$, developed from the log likelihood function $\mathbf{L}(\mathbf{F}(\hat{\mathbf{S}}) \mid \mathbf{F}(\mathbf{X}), \mathbf{A})$ with the assumption of Laplanicity of source signals in the Fourier domain

following the probability $\mathbf{P(F(S))} = \frac{\boldsymbol{\lambda}}{\mathbf{2}} e^{-\lambda \mathbf{c}^{\mathrm{T}} |\mathbf{F}(\hat{\mathbf{S}})|}$, where $\mathbf{c}^{\mathrm{T}} = [1, 1, \ldots 1]$ is a unit vector, with the convergence criteria, min $\boldsymbol{\lambda}\mathbf{c}^{\mathrm{T}}\left|\mathbf{F}(\hat{\mathbf{S}})\right|$, evaluated based on the clustered mixed signal samples to determine whether the convergence criteria, min $\boldsymbol{\lambda}\mathbf{c}^{\mathrm{T}}\left|\mathbf{F}(\hat{\mathbf{S}})\right|$, is met, and if the convergence criteria, min $\boldsymbol{\lambda}\mathbf{c}^{\mathrm{T}}\left|\mathbf{F}(\hat{\mathbf{S}})\right|$, is not met, iteratively adjusting the clustering of the mixed signal samples and parameters of the geometric constraint to create a new set of clusters until the convergence criteria, min $\boldsymbol{\lambda}\mathbf{c}^{\mathrm{T}}\left|\mathbf{F}(\hat{\mathbf{S}})\right|$, is met, to provide a final estimated mixing matrix $\hat{\mathbf{A}}$.

20. A method for blind separation of an overcomplete set of mixed signals as set forth in claim 19, wherein the wherein the step of jointly optimizing the source signal estimate matrix $\hat{\mathbf{S}}$ and the estimated mixing matrix $\hat{\mathbf{A}}$ in an iterative manner, to generate an optimized source signal estimate matrix $\hat{\mathbf{S}}$ and a final estimated mixing matrix $\hat{\mathbf{A}}$ further comprises the sub steps of:

i. obtaining a multi-band sparse domain estimate of the source signal estimate matrix $\hat{\mathbf{S}}$ using the relationship $\mathbf{X} = \hat{\mathbf{A}}\hat{\mathbf{S}} + \mathbf{V}$, applied in the Wavelet domain; and

ii. using the adjusted geometric constraint corresponding to the final estimated mixing matrix $\hat{\mathbf{A}}$ in each of the bands of the Wavelet domain for the source signal estimate matrix $\hat{\mathbf{S}}$, $\mathbf{W}(\hat{\mathbf{S}})$, and determining whether a convergence criteria, min $\boldsymbol{\lambda}\mathbf{c}^{\mathrm{T}}\left|\mathbf{W}(\hat{\mathbf{S}})\right|$ is met for the source signal estimate matrix $\hat{\mathbf{S}}$, where the convergence criteria, min $\boldsymbol{\lambda}\mathbf{c}^{\mathrm{T}}\left|\mathbf{W}(\hat{\mathbf{S}})\right|$, is developed from the log likelihood function $\mathbf{L}(\mathbf{W}(\hat{\mathbf{S}}) \mid \mathbf{W}(\mathbf{X}), \mathbf{A})$ with the assumption of Laplanicity of source signals in the Wavelet domain following the probability $\mathbf{P(W(S))} = \frac{\boldsymbol{\lambda}}{\mathbf{2}} e^{-\lambda \mathbf{c}^{\mathrm{T}} |\mathbf{W}(\hat{\mathbf{S}})|}$, where $\mathbf{c}^{\mathrm{T}} = [1, 1, \ldots 1]$ is a unit vector, and if the convergence criteria is not met,

$\min \boldsymbol{\lambda}\mathbf{c}^{T}\left|\mathbf{W}(\hat{\mathbf{S}})\right|$, iteratively adjusting the clustering of the mixed signal samples to create a new set of clusters until the convergence criteria, $\min \boldsymbol{\lambda}\mathbf{c}^{T}\left|\mathbf{W}(\hat{\mathbf{S}})\right|$, is met, to provide a final source signal estimate matrix $\hat{\mathbf{S}}$.

21. A method for blind separation of an overcomplete set of mixed signals as set forth in claim 20, wherein the method is configured to separate mixed acoustic signals.

22. A method for blind separation of an overcomplete set of mixed signals as set forth in claim 20, wherein the method is configured to separate mixed radio frequency signals.

23. A computer program product for blind separation of an overcomplete set of mixed signals, readable on a data processing system including an input for receiving mixed signals from a plurality of sensors configured to receive mixed signal samples comprising a mixture of signals transmitted from signal sources through an environment and noise, a signal processor attached with the input for receiving the mixed signals from the sensors, and a memory for storing data during operations of the signal processor the computer program product comprising means, stored on a computer readable medium, for:

   i. storing data representing the input from the sensors in a mixed signal matrix $\mathbf{X}$;
   ii. storing data representing the noise in a noise matrix $\mathbf{V}$;
   iii. storing data representing an estimate of the individual signals from the mixture of signals from the signal sources in a source signal estimate matrix $\hat{\mathbf{S}}$;
   iv. storing data representing an estimate of the effects of the environment in a estimated mixing matrix $\hat{\mathbf{A}}$ where the matrices are related by $\mathbf{X} = \hat{\mathbf{A}}\hat{\mathbf{S}} + \mathbf{V}$;
   v. generating an initial estimate of the estimated mixing matrix $\hat{\mathbf{A}}$;

vi. determining the number of signal sources and associated lines of correlation of each of the signal sources from the estimated mixing matrix $\hat{\mathbf{A}}$, and for representing the signal sources in the source signal estimate matrix $\hat{\mathbf{S}}$;

vii. jointly optimizing the source signal estimate matrix $\hat{\mathbf{S}}$ and the estimated mixing matrix $\hat{\mathbf{A}}$ in an iterative manner, to generate an optimized source signal estimate matrix $\hat{\mathbf{S}}$ and a final estimated mixing matrix $\hat{\mathbf{A}}$; and

viii. restoring the separated source signals from the optimized source signal estimate matrix $\hat{\mathbf{S}}$, whereby a plurality of mixed signals from unknown sources traveling through an environment with added noise may be separated so that the original, separate signals may be reconstructed.

24. A computer program product for blind separation of an overcomplete set of mixed signals as set forth in claim 23, wherein the means for generating an initial estimate of the estimated mixing matrix $\hat{\mathbf{A}}$ comprises:

i. means for transforming the mixed signal matrix $\mathbf{X}$ into the sparse domain using a transform operator;

ii. means for determining a frequency band within the sparse domain that contains the most information that can be used to determine lines of correlation to determine the number of signal sources;

iii. means for determining a measure and an optimal threshold for the measure for the determination of noise within the frequency band;

iv. means for recalculating the measure used in the determination of the noise within the frequency band using the optimal threshold; and

v. means for determining the local maxima of a distribution of the measure, where the local maxima represent angles which are inserted into the estimated mixing matrix $\hat{\mathbf{A}}$ to provide an initial estimate of the estimated mixing matrix $\hat{\mathbf{A}}$.

25. A computer program product for blind separation of an overcomplete set of mixed signals as set forth in claim 24, wherein the means for jointly optimizing the source

signal estimate matrix $\hat{\mathbf{S}}$ and the estimated mixing matrix $\hat{\mathbf{A}}$ in an iterative manner, to generate an optimized source signal estimate matrix $\hat{\mathbf{S}}$ and a final estimated mixing matrix $\hat{\mathbf{A}}$ comprises:

i. means for clustering the mixed signal samples using a geometric constraint; and

ii. means for evaluating a convergence criteria based on the clustered mixed signal samples to determine whether the convergence criteria are met, and if the convergence criteria are not met, iteratively adjusting the clustering of the mixed signal samples and parameters of the geometric constraint to create a new set of clusters until the convergence criteria are met, to provide a final estimated mixing matrix $\hat{\mathbf{A}}$.

26. A computer program product for blind separation of an overcomplete set of mixed signals as set forth in claim 25, wherein the means for transforming the mixed signal matrix **X** into the sparse domain using a transform operator is a Fourier transform operator such that the estimated mixing matrix $\hat{\mathbf{A}}$ is represented in the Fourier transform, and wherein the means for obtaining a multi-band sparse domain estimate of the source signal estimate matrix $\hat{\mathbf{S}}$ using the relationship $\mathbf{X} = \hat{\mathbf{A}}\hat{\mathbf{S}} + \mathbf{V}$ uses a wavelet transform operator to obtain the multi-band sparse domain estimate.

27. A computer program product for blind separation of an overcomplete set of mixed signals as set forth in claim 24, wherein the wherein the means for jointly optimizing the source signal estimate matrix $\hat{\mathbf{S}}$ and the estimated mixing matrix $\hat{\mathbf{A}}$ in an iterative manner, to generate an optimized source signal estimate matrix $\hat{\mathbf{S}}$ and a final estimated mixing matrix $\hat{\mathbf{A}}$ further comprises:

i. means for obtaining a multi-band sparse domain estimate of the source signal estimate matrix $\hat{\mathbf{S}}$ using the relationship $\mathbf{X} = \hat{\mathbf{A}}\hat{\mathbf{S}} + \mathbf{V}$; and

ii. means for using the adjusted geometric constraint corresponding to the final estimated mixing matrix $\hat{\mathbf{A}}$ in each of the bands of the sparse domain for the source signal estimate matrix $\hat{\mathbf{S}}$ and determining whether a convergence criteria

is met for the source signal estimate matrix $\hat{\mathbf{S}}$, and if the convergence criteria are not met, iteratively adjusting the clustering of the mixed signal samples to create a new set of clusters until the convergence criteria are met, to provide a final source signal estimate matrix $\hat{\mathbf{S}}$.

28. A computer program product for blind separation of an overcomplete set of mixed signals as set forth in claim 27, wherein the means for transforming the mixed signal matrix **X** into the sparse domain using a transform operator is a Fourier transform operator such that the estimated mixing matrix $\hat{\mathbf{A}}$ is represented in the Fourier transform, and wherein the means for obtaining a multi-band sparse domain estimate of the source signal estimate matrix $\hat{\mathbf{S}}$ using the relationship $\mathbf{X} = \hat{\mathbf{A}}\hat{\mathbf{S}} + \mathbf{V}$ uses a wavelet transform operator to obtain the multi-band sparse domain estimate.

29. A computer program product for blind separation of an overcomplete set of mixed signals as set forth in claim 23, wherein the means for generating an initial estimate of the estimated mixing matrix $\hat{\mathbf{A}}$ comprises:

i. means for transforming the mixed signal matrix **X** into the frequency domain using a Fourier operator;

ii. means for using a mutual information criterion to determine a frequency band within the sparse domain that contains the most information that can be used to determine lines of correlation to determine the number of signal sources;

iii. means for determining a random variable $ang = \arctan\frac{x_i(band)}{x_j(band)}$, where $x_i(band)$ and $x_j(band)$ represent Fourier values of mixture in the selected frequency band, and an optimal threshold ANG for *ang*, where the optimal threshold ANG is determined by computing the entropy E(*ang*, ANG) vs. ANG and searching for the optimal value of ANG corresponding to the minimum rate of descent of the entropy E(*ang*, ANG);

iv. means for recalculating *ang* based on the optimal threshold ANG;

v. means for using a standard peak detection technique to determine the number and values of local maxima of a histogram of *ang* where the local maxima represent angles which are inserted into the estimated mixing matrix $\hat{\mathbf{A}}$ to provide an initial estimate of the estimated mixing matrix $\hat{\mathbf{A}}$.

30. A computer program product for blind separation of an overcomplete set of mixed signals as set forth in claim 29, wherein the means for jointly optimizing the source signal estimate matrix $\hat{\mathbf{S}}$ and the estimated mixing matrix $\hat{\mathbf{A}}$ in an iterative manner, to generate an optimized source signal estimate matrix $\hat{\mathbf{S}}$ and a final estimated mixing matrix $\hat{\mathbf{A}}$ comprises:

i. means for clustering the mixed signal samples in the Fourier domain along the lines of correlation with one cluster per source using a straight distance metric geometric constraint, with the clusters representing estimates of the Fourier domain representation of $\hat{\mathbf{S}}$, $\mathbf{F}(\hat{\mathbf{S}})$, where $\mathbf{F}$ represents a Fourier domain operator ; and

ii. means for evaluating a convergence criteria, min $\boldsymbol{\lambda}\mathbf{c}^{\mathbf{T}}\left|\mathbf{F}(\hat{\mathbf{S}})\right|$, with the convergence criteria, min $\boldsymbol{\lambda}\mathbf{c}^{\mathbf{T}}\left|\mathbf{F}(\hat{\mathbf{S}})\right|$, developed from the log likelihood function $\mathbf{L}(\mathbf{F}(\hat{\mathbf{S}}) \mid \mathbf{F}(\mathbf{X}), \mathbf{A})$ with the assumption of Laplanicity of source signals in the Fourier domain following the probability $\mathbf{P}(\mathbf{F}(\mathbf{S})) = \frac{\boldsymbol{\lambda}}{\mathbf{2}} e^{-\boldsymbol{\lambda}\mathbf{c}^{\mathbf{T}}\left|\mathbf{F}(\hat{\mathbf{S}})\right|}$, where $\mathbf{c}^{\mathbf{T}} = [1, 1, \ldots 1]$ is a unit vector, with the convergence criteria, min $\boldsymbol{\lambda}\mathbf{c}^{\mathbf{T}}\left|\mathbf{F}(\hat{\mathbf{S}})\right|$, evaluated based on the clustered mixed signal samples to determine whether the convergence criteria, min $\boldsymbol{\lambda}\mathbf{c}^{\mathbf{T}}\left|\mathbf{F}(\hat{\mathbf{S}})\right|$, is met, and if the convergence criteria, min $\boldsymbol{\lambda}\mathbf{c}^{\mathbf{T}}\left|\mathbf{F}(\hat{\mathbf{S}})\right|$, is not met, iteratively adjusting the clustering of the mixed signal samples and parameters of the geometric constraint to create a new set of clusters until the convergence criteria, min $\boldsymbol{\lambda}\mathbf{c}^{\mathbf{T}}\left|\mathbf{F}(\hat{\mathbf{S}})\right|$, is met, to provide a final estimated mixing matrix $\hat{\mathbf{A}}$.

31. A computer program product for blind separation of an overcomplete set of mixed signals as set forth in claim 30, wherein the wherein the means for jointly optimizing the source signal estimate matrix $\hat{\mathbf{S}}$ and the estimated mixing matrix $\hat{\mathbf{A}}$ in an iterative manner, to generate an optimized source signal estimate matrix $\hat{\mathbf{S}}$ and a final estimated mixing matrix $\hat{\mathbf{A}}$ further comprises:

i. means for obtaining a multi-band sparse domain estimate of the source signal estimate matrix $\hat{\mathbf{S}}$ using the relationship $\mathbf{X} = \hat{\mathbf{A}}\hat{\mathbf{S}} + \mathbf{V}$, applied in the Wavelet domain; and

ii. means for using the adjusted geometric constraint corresponding to the final estimated mixing matrix $\hat{\mathbf{A}}$ in each of the bands of the Wavelet domain for the source signal estimate matrix $\hat{\mathbf{S}}$, $\mathbf{W}(\hat{\mathbf{S}})$, and determining whether a convergence criteria, min $\boldsymbol{\lambda}\mathbf{c}^{\mathrm{T}}\left|\mathbf{W}(\hat{\mathbf{S}})\right|$ is met for the source signal estimate matrix $\hat{\mathbf{S}}$, where the convergence criteria, min $\boldsymbol{\lambda}\mathbf{c}^{\mathrm{T}}\left|\mathbf{W}(\hat{\mathbf{S}})\right|$, is developed from the log likelihood function $\mathbf{L}(\mathbf{W}(\hat{\mathbf{S}}) \mid \mathbf{W}(\mathbf{X}), \mathbf{A})$ with the assumption of Laplanicity of source signals in the Wavelet domain following the probability $\mathbf{P}(\mathbf{W}(\mathbf{S})) = \frac{\boldsymbol{\lambda}}{\mathbf{2}} e^{-\lambda \mathbf{c}^{\mathrm{T}}\left|\mathbf{W}(\hat{\mathbf{S}})\right|}$, where $\mathbf{c}^{\mathrm{T}} = [1, 1, \ldots 1]$ is a unit vector, and if the convergence criteria is not met, min $\boldsymbol{\lambda}\mathbf{c}^{\mathrm{T}}\left|\mathbf{W}(\hat{\mathbf{S}})\right|$, iteratively adjusting the clustering of the mixed signal samples to create a new set of clusters until the convergence criteria, min $\boldsymbol{\lambda}\mathbf{c}^{\mathrm{T}}\left|\mathbf{W}(\hat{\mathbf{S}})\right|$, is met, to provide a final source signal estimate matrix $\hat{\mathbf{S}}$.

32. A computer program product for blind separation of an overcomplete set of mixed signals as set forth in claim 30, wherein the computer program product is configured for separating mixed acoustic signals.

33. A computer program product for blind separation of an overcomplete set of mixed signals as set forth in claim 30, wherein the computer program product is configured for separating mixed radio frequency signals.

34. An apparatus for determining a CR bound for an estimated mixing matrix $\hat{\mathbf{A}}$ developed in the blind separation of an overcomplete set of mixed signals, the apparatus comprising a data processing system including a processor, a memory coupled with the processor, an input coupled with the processor, an output coupled with the processor, means within the data processing system for generating a CR bound for the estimated mixing matrix $\hat{\mathbf{A}}$, and means for generating an output of the expected value for the estimation error of associated lines of correlation and for providing the output to a user via the output, whereby a CR bound may be developed for determining the performance of an estimate of a mixing matrix $\hat{\mathbf{A}}$ developed in the blind separation of an overcomplete set of mixed signals in order that a user may know the performance limitations of a blind separation apparatus.

35. An apparatus for determining a CR bound for an estimated mixing matrix $\hat{\mathbf{A}}$ developed in the blind separation of an overcomplete set of mixed signals as set forth in claim 34, wherein the means for determining the expected value for the estimation error is in the form of $E\left\{\left(\theta_i - \hat{\theta}_i\right)^2\right\}$ where

$E\left\{\left(\theta_i - \hat{\theta}_i\right)^2\right\} \geq \dfrac{\lambda_{\mathbf{k}}^2}{2N\mathbf{u}^T(\theta_i)\mathbf{p}^T\mathbf{R}_{W(V)}^{-1}\mathbf{p}\mathbf{u}(\theta_i)}$, where:

$E\left\{\left(\theta_i - \hat{\theta}_i\right)^2\right\}$ is an expected value for the estimation error of associated lines of correlation;

$\theta_i = \arctan\left(\dfrac{\mathbf{a}_i}{\|\mathbf{a}_i\|}\right)$, where $\mathbf{a}_i = \begin{bmatrix} a_{1i} \\ a_{2i} \end{bmatrix}, i = 1, 2, \ldots M$, and $\hat{\mathbf{A}} = \hat{\mathbf{A}}(\theta) = \mathbf{u}(\theta_i)$;

$\hat{\theta}_i$ is an estimated value corresponding to an actual value of θ_i;

λ_k^2 is developed from the log likelihood function $L(\mathbf{W}(\hat{\mathbf{S}}) \mid \mathbf{W}(\mathbf{X}), \mathbf{A}(\theta))$ used for the estimation of the mixing matrix $\hat{\mathbf{A}}$ and the estimation of a source signal estimate matrix $\hat{\mathbf{S}}$;

N is a number of data samples used in the generation of the mixing matrix $\hat{\mathbf{A}}$ and the source signal estimate matrix $\hat{\mathbf{S}}$;

$\mathbf{u}(\theta_i) = \begin{bmatrix} \cos(\theta_i) \\ \sin(\theta_i) \end{bmatrix}$;

$\mathbf{p} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$;

T is the transpose operator; and

$\mathbf{R}_{W(V)}^{-1} = \begin{bmatrix} \sigma_{\mathrm{W(V)}}^2 & \rho\sigma_{\mathrm{W(V)}}^2 \\ \rho\sigma_{\mathrm{W(V)}}^2 & \sigma_{\mathrm{W(V)}}^2 \end{bmatrix}$, where $\sigma_{\mathrm{W(V)}}^2$ is a cross correlation of a noise set and ρ is a constant multiplier value.

36. An apparatus for determining a CR bound for an source signal estimate matrix $\hat{\mathbf{S}}$ developed in the blind separation of an overcomplete set of mixed signals, the apparatus comprising a data processing system including a processor, a memory coupled with the processor, an input coupled with the processor, an output coupled with the processor, means within the data processing system for generating a CR bound for the source signal estimate matrix $\hat{\mathbf{S}}$, and means for generating an output of the expected value for the estimation error of associated lines of correlation and for providing the output to a user via the output, whereby a CR bound may be developed for determining the performance of an estimate of a source signal estimate matrix $\hat{\mathbf{S}}$ developed in the blind separation of an overcomplete set of mixed signals in order that a user may know the performance limitations of a blind separation apparatus.

37. An apparatus for determining a CR bound for a source signal estimate matrix $\hat{\mathbf{S}}$ developed in the blind separation of an overcomplete set of mixed signals as set forth in claim 36, wherein the means for determining the expected value for the estimation

error is in the form of $E\left\{(\mathbf{W}(\mathbf{S})-\mathbf{W}(\hat{\mathbf{S}}))^2\right\}$

where $E\left\{(\mathbf{W}(\mathbf{S})-\mathbf{W}(\hat{\mathbf{S}}))^2\right\} \geq \left(\sigma_v^2 \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \hat{\mathbf{A}}^T(\theta)\hat{\mathbf{A}}(\theta) + \lambda^2 \mathbf{I}\right)^{-1}$, where

σ_v^2 represents a noise level;

ρ is a constant multiplier value;

$\hat{\mathbf{A}}$ is an estimated mixing matrix;

$\theta_i = \arctan\left(\frac{\mathbf{a}_i}{\|\mathbf{a}_i\|}\right)$, where $\mathbf{a}_i = \begin{bmatrix} a_{1i} \\ a_{2i} \end{bmatrix}, i = 1, 2, \ldots M$, and $\hat{\mathbf{A}} = \hat{\mathbf{A}}(\theta) = \mathbf{u}(\theta_i)$;

$\lambda_{\mathbf{k}}^2$ is developed from the log likelihood function $L(\mathbf{W}(\hat{\mathbf{S}}) \mid \mathbf{W}(\mathbf{X}), \mathbf{A}(\theta))$; and

I is an identity matrix.

38. A method for determining a CR bound for an estimated mixing matrix $\hat{\mathbf{A}}$ developed in the blind separation of an overcomplete set of mixed signals, operating on an apparatus comprising a data processing system including a processor, a memory coupled with the processor, an input coupled with the processor, an output coupled with the processor, the method comprising the steps of generating a CR bound for the estimated mixing matrix $\hat{\mathbf{A}}$, and generating an output of the expected value for the estimation error of associated lines of correlation and for providing the output to a user via the output, whereby a CR bound may be developed for determining the performance of an estimate of a mixing matrix $\hat{\mathbf{A}}$ developed in the blind separation of an overcomplete set of mixed signals in order that a user may know the performance limitations of a blind separation apparatus.

39. A method for determining a CR bound for an estimated mixing matrix $\hat{\mathbf{A}}$ developed in the blind separation of an overcomplete set of mixed signals as set forth in claim 38, wherein in the step of determining the expected value for the estimation error, the

expected value for estimation error is in the form of $E\left\{\left(\theta_i - \hat{\theta}_i\right)^2\right\}$ where

$$E\left\{\left(\theta_i - \hat{\theta}_i\right)^2\right\} \geq \frac{\lambda_k^2}{2N\mathbf{u}^T(\theta_i)\mathbf{p}^T\mathbf{R}_{W(V)}^{-1}\mathbf{p}\mathbf{u}(\theta_i)}, \text{ where:}$$

$E\left\{\left(\theta_i - \hat{\theta}_i\right)^2\right\}$ is an expected value for the estimation error of associated lines of correlation;

$\theta_i = \arctan\left(\frac{\mathbf{a}_i}{\|\mathbf{a}_i\|}\right)$, where $\mathbf{a}_i = \begin{bmatrix} a_{1i} \\ a_{2i} \end{bmatrix}, i = 1, 2, \ldots M$, and $\hat{\mathbf{A}} = \hat{\mathbf{A}}(\theta) = \mathbf{u}(\theta_i)$;

$\hat{\theta}_i$ is an estimated value corresponding to an actual value of θ_i;

λ_k^2 is developed from the log likelihood function $L(\mathbf{W}(\hat{\mathbf{S}}) \mid \mathbf{W}(\mathbf{X}), \mathbf{A}(\theta))$ used for the estimation of the mixing matrix $\hat{\mathbf{A}}$ and the estimation of a source signal estimate matrix $\hat{\mathbf{S}}$;

N is a number of data samples used in the generation of the mixing matrix $\hat{\mathbf{A}}$ and the source signal estimate matrix $\hat{\mathbf{S}}$;

$\mathbf{u}(\theta_i) = \begin{bmatrix} \cos(\theta_i) \\ \sin(\theta_i) \end{bmatrix}$;

$\mathbf{p} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$;

T is the transpose operator; and

$\mathbf{R}_{W(V)}^{-1} = \begin{bmatrix} \sigma_{\mathrm{W(V)}}^2 & \rho\sigma_{\mathrm{W(V)}}^2 \\ \rho\sigma_{\mathrm{W(V)}}^2 & \sigma_{\mathrm{W(V)}}^2 \end{bmatrix}$, where $\sigma_{\mathrm{W(V)}}^2$ is a cross correlation of a noise set and ρ is a constant multiplier value.

40. A method for determining a CR bound for an source signal estimate matrix $\hat{\mathbf{S}}$ developed in the blind separation of an overcomplete set of mixed signals, operated in an apparatus comprising a data processing system including a processor, a memory coupled with the processor, an input coupled with the processor, an output coupled with the processor, the method comprising the steps of generating a CR bound for the source signal estimate matrix $\hat{\mathbf{S}}$, and generating an output of the expected value for

the estimation error of associated lines of correlation and for providing the output to a user via the output, whereby a CR bound may be developed for determining the performance of an estimate of a source signal estimate matrix $\hat{\mathbf{S}}$ developed in the blind separation of an overcomplete set of mixed signals in order that a user may know the performance limitations of a blind separation apparatus.

41. A method of determining a CR bound for a source signal estimate matrix $\hat{\mathbf{S}}$ developed in the blind separation of an overcomplete set of mixed signals as set forth in claim 40, wherein the in the step of determining the expected value for the estimation error, the expected value for the estimation error is in the form of $E\left\{\left(\mathbf{W}(\mathbf{S})-\mathbf{W}(\hat{\mathbf{S}})\right)^2\right\}$ where $E\left\{\left(\mathbf{W}(\mathbf{S})-\mathbf{W}(\hat{\mathbf{S}})\right)^2\right\} \geq \left(\sigma_v^2\begin{bmatrix} 1 & \rho \\ \rho & 1\end{bmatrix}\hat{\mathbf{A}}^T(\theta)\hat{\mathbf{A}}(\theta)+\lambda^2\mathbf{I}\right)^{-1}$,

where

σ_v^2 represents a noise level;

ρ is a constant multiplier value;

$\hat{\mathbf{A}}$ is an estimated mixing matrix;

$\theta_i = \arctan\left(\frac{\mathbf{a}_i}{\|\mathbf{a}_i\|}\right)$, where $\mathbf{a}_i = \begin{bmatrix} a_{1i} \\ a_{2i}\end{bmatrix}, i = 1, 2, \ldots M$, and $\hat{\mathbf{A}} = \hat{\mathbf{A}}(\theta) = \mathbf{u}(\theta_i)$;

$\lambda_{\mathbf{k}}^2$ is developed from the log likelihood function $L(\mathbf{W}(\hat{\mathbf{S}}) \mid \mathbf{W}(\mathbf{X}), \mathbf{A}(\theta))$; and

$\mathbf{I}$ is an identity matrix.

42. A computer program product for determining a CR bound for an estimated mixing matrix $\hat{\mathbf{A}}$ developed in the blind separation of an overcomplete set of mixed signals, the computer program product being written onto a medium readable on a data processing system including a processor, a memory coupled with the processor, an input coupled with the processor, an output coupled with the processor, with the computer program product comprising means for generating a CR bound for the estimated mixing matrix $\hat{\mathbf{A}}$, and means for generating an output of the expected value for the estimation error of associated lines of correlation and for providing the

output to a user via the output, whereby a CR bound may be developed for determining the performance of an estimate of a mixing matrix $\hat{\mathbf{A}}$ developed in the blind separation of an overcomplete set of mixed signals in order that a user may know the performance limitations of a blind separation apparatus.

43. A computer program product for determining a CR bound for an estimated mixing matrix $\hat{\mathbf{A}}$ developed in the blind separation of an overcomplete set of mixed signals as set forth in claim 42, wherein the means for determining the expected value for the estimation error determines an estimation error by calculating $E\left\{\left(\theta_i - \hat{\theta}_i\right)^2\right\}$ where

$$E\left\{\left(\theta_i - \hat{\theta}_i\right)^2\right\} \geq \frac{\lambda_{\mathbf{k}}^2}{2N\mathbf{u}^T(\theta_i)\mathbf{p}^T\mathbf{R}_{W(V)}^{-1}\mathbf{p}\mathbf{u}(\theta_i)},$$ where:

$E\left\{\left(\theta_i - \hat{\theta}_i\right)^2\right\}$ is an expected value for the estimation error of associated lines of correlation;

$\theta_i = \arctan\left(\frac{\mathbf{a}_i}{\|\mathbf{a}_i\|}\right)$, where $\mathbf{a}_i = \begin{bmatrix} a_{1i} \\ a_{2i} \end{bmatrix}, i = 1, 2, \ldots M$, and $\hat{\mathbf{A}} = \hat{\mathbf{A}}(\theta) = \mathbf{u}(\theta_i)$;

$\hat{\theta}_i$ is an estimated value corresponding to an actual value of θ_i;

$\lambda_{\mathbf{k}}^2$ is developed from the log likelihood function $L(\mathbf{W}(\hat{\mathbf{S}}) \mid \mathbf{W}(\mathbf{X}), \mathbf{A}(\theta))$ used for the estimation of the mixing matrix $\hat{\mathbf{A}}$ and the estimation of a source signal estimate matrix $\hat{\mathbf{S}}$;

N is a number of data samples used in the generation of the mixing matrix $\hat{\mathbf{A}}$ and the source signal estimate matrix $\hat{\mathbf{S}}$;

$\mathbf{u}(\theta_i) = \begin{bmatrix} \cos(\theta_i) \\ \sin(\theta_i) \end{bmatrix}$;

$\mathbf{p} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$;

T is the transpose operator; and

$$\mathbf{R}_{W(V)}^{-1} = \begin{bmatrix} \sigma_{\mathbf{W(V)}}^2 & \rho\sigma_{\mathbf{W(V)}}^2 \\ \rho\sigma_{\mathbf{W(V)}}^2 & \sigma_{\mathbf{W(V)}}^2 \end{bmatrix}$$, where $\sigma_{\mathbf{W(V)}}^2$ is a cross correlation of a noise set and ρ is a constant multiplier value.

44. A computer program product for determining a CR bound for an source signal estimate matrix $\hat{\mathbf{S}}$ developed in the blind separation of an overcomplete set of mixed signals, the computer program product being written onto a medium readable on a data processing system including a processor, a memory coupled with the processor, an input coupled with the processor, an output coupled with the processor, with the computer program product comprising means for generating a CR bound for the source signal estimate matrix $\hat{\mathbf{S}}$, and means for generating an output of the expected value for the estimation error of associated lines of correlation and for providing the output to a user via the output, whereby a CR bound may be developed for determining the performance of an estimate of a source signal estimate matrix $\hat{\mathbf{S}}$ developed in the blind separation of an overcomplete set of mixed signals in order that a user may know the performance limitations of a blind separation apparatus.

45. A computer program product for determining a CR bound for a source signal estimate matrix $\hat{\mathbf{S}}$ developed in the blind separation of an overcomplete set of mixed signals as set forth in claim 44, wherein the means for determining the expected value for the estimation error determines an estimation error by calculating $E\left\{(\mathbf{W(S)} - \mathbf{W(\hat{S})})^2\right\}$

where $E\left\{(\mathbf{W(S)} - \mathbf{W(\hat{S})})^2\right\} \geq \left(\sigma_v^2 \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \hat{\mathbf{A}}^T(\theta)\hat{\mathbf{A}}(\theta) + \lambda^2\mathbf{I}\right)^{-1}$, where

σ_v^2 represents a noise level;

ρ is a constant multiplier value;

$\hat{\mathbf{A}}$ is an estimated mixing matrix;

$\theta_i = \arctan\left(\frac{\mathbf{a}_i}{\|\mathbf{a}_i\|}\right)$, where $\mathbf{a}_i = \begin{bmatrix} a_{1i} \\ a_{2i} \end{bmatrix}, i = 1, 2, \ldots M$, and $\hat{\mathbf{A}} = \hat{\mathbf{A}}(\theta) = \mathbf{u}(\theta_i)$;

$\lambda_{\mathbf{k}}^{2}$ is developed from the log likelihood function $L(\mathbf{W}(\hat{\mathbf{S}}) \mid \mathbf{W}(\mathbf{X}), \mathbf{A}(\theta))$; and

I is an identity matrix.